TOPOLOGY 2
FINAL EXAMINATION

This exam is of **50 marks**. Please **read all the questions carefully** and **do not cheat**. Please feel free to use whatever theorems you have learned in class after stating them clearly. Please **hand in your phones** at the beginning of the class. Please sign the following statement:

I have not used any unfair or illegal means to answer any of the questions in this exam. I have answered this paper honestly and truthfully.

Name: **Signature:**

1. Let X be the graph known as the *Moser Spindle*:

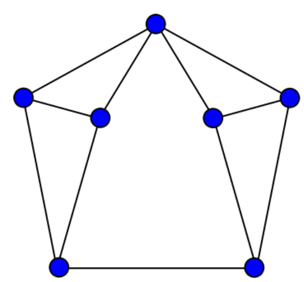

FIGURE 1. Moser Spindle

a. Compute the fundamental group $\pi_1(X, P)$ for some vertex P on X. (5)

b. Compute the homology groups $H_i(X, \mathbb{Z})$ for all i. (10)

c. Compute the **Betti numbers** and the **Euler characteristic** $\chi(X)$. (3,2)

d. Extra Credit - Why has this been in the news lately? (1)

2. Prove the **Five Lemma**: If A_i, B_i $1 \leq i \leq 5$ are R modules with homomorphisms $f_i : A_i \to A_{i+1}$ and $g_i : B_i \to B_{i+1}$ and homomorphisms $\alpha, \beta, \gamma, \delta$ and ϵ from $A_i \to B_i$ such that in the diagram the **rows are exact**, α is **surjective**, ϵ is **injective** and β and δ are **isomorphisms**, then γ is an **isomorphism**.

$$\begin{array}{ccccccccc} A_1 & \xrightarrow{f_1} & A_2 & \xrightarrow{f_2} & A_3 & \xrightarrow{f_3} & A_4 & \xrightarrow{f_4} & A_5 \\ \alpha\downarrow & & \beta\downarrow & & \gamma\downarrow & & \delta\downarrow & & \epsilon\downarrow \\ B_1 & \xrightarrow{g_1} & B_2 & \xrightarrow{g_2} & B_3 & \xrightarrow{g_3} & B_4 & \xrightarrow{g_4} & B_5 \end{array}$$

3a. State the **Excision Theorem** (3)

3b. Use it to prove that (7)

$$H_q(S^n, \mathbb{Z}) \simeq H_{q-1}(S^{n-1}, \mathbb{Z})$$

for $q \geq 2$, $n \geq 1$.

4. Let $T = \mathbb{C}/\mathbb{Z}^2$ be the torus

4a. Compute the fundamental group $\pi_1(T, \bar{i})$, where $i = \sqrt{-1}$ and $\bar{i}$ denotes its image under the canonical map. (4)

4b. Compute the homology groups $H_i(T, \mathbb{Z})$ for all i. (6)